

Comments on the March 2010 draft of the Common Core Standards  
by Bert Fristedt

I was a member of the National Mathematics Advisory Panel. Within that Panel I was on the Assessment Task Group and the Instructional Materials Subcommittee. I supported the reports of all three entities. However, my remarks here are mine alone; they should not be construed as representing the views of any other member of the Panel.

Some of my comments will refer to specific items in the draft; I only bring such items into my comments when, in my judgment, they represent a general feature or concern with Common Core.

The following will be the most important readers of the Common Core Document:

1. Textbook writers and teachers,
2. Broadly given assessment constructors,
3. School and state education administrators.

I call you attention to the fourteen items for Grade 3 where multiplication or division is significantly involved: Items 1-9 under 'Number—Operations and the Problems They Solve' on page 19; Items 7-9 on page 20 under the heading 'Number—Base Ten'; Item 3 under 'Measurement and Data' on page 20; and Item 4 under 'Geometry' on page 21. Consider the dilemmas facing the textbook writers, teachers and those creating NAEP or other such assessments. Item 8 on page 20 is fine and grade-appropriate. On the other hand, asking a third grade student to do the explaining connected with Item 7 on the same page would be an unreasonably high expectation. And comparing this item with Item 9 on the same page gives the impression that students, at the Grade 3 level, need not memorize that  $6 \times 8 = 48$  but that, nevertheless, they should be able to calculate it using the distributive law, presumably even when it arises in the middle of a problem related to Item 4 on page 21 or Items 7 and 8 on Page 19. The prevalence of 'Understand' items on page 19 —7 out of 9, including two with several parts is very disturbing. A teacher could not be blamed—indeed, might be lauded by a principal who has read the Common Core Standards—for using class time in a fragmented, unfocused manner, talking endlessly (for a listening third-grader) about the principles of multiplication and division.

Writers of third grade materials, desirous of being consistent with the Common Core Standards, might in extreme cases use section and subsection headings from the fourteen items identified in the above paragraph, thus creating an unbalanced view of expectations for third graders. Even if they do not go to that extreme, a fragmented, overly wordy text with too few grade-appropriate examples and exercises could easily be the result. An author who is willing to ignore Common Core, but instead to think through multiplication and division in the spirit of the 'Focal Points' document of the National Council of Teachers of Mathematics is likely to write a more coherent, student-friendly, and focused

text. The multiplication and division issues at Grade 4 are somewhat similar, but at a more advanced level. There are multiplication and division items in the Common Core Standards for these two grades that are consistent with the automaticity described by the Learning Processes Task Group of the National Mathematics Advisory Panel, but the presentation in the Standards dilutes focus and thus makes achievement of the desired automaticity more difficult than it should be.

And I can envision different constructors of broadly given assessments arriving at far different conclusions about what can be assessed at the third-grade level consistent with the Common Core Standards, thus rendering the adjective ‘common’ somewhat meaningless.

The varying levels of specificity, lack of clear flow from grade to grade, and missing connections within grades has resulted in a document that can be read with very different understanding by different textbook writers and teachers—and also by those who construct broadly given assessments. Example: Even though  $a^3 = a \times a \times a$  is mentioned for grade 7 (page 36), understanding positive integer exponents does not appear until high school (page 44), and even there insufficient emphasis is given to what a student should be able to do—say calculating  $3^5$  by multiplying 27 and 9. The laws of exponents mentioned under ‘The Real Number System’ line of page 44 are relevant to scientific notation described under the different heading ‘Quantities’ on the same page, but the connection is not mentioned. More generally, a textbook publisher will be puzzled on how to include exponents for a sequence of grades in a clear-flowing coherent manner.

The fact that Common Core is freed from the strand structure that seemed to govern state standards, enables Common Core to adopt headings to highlight appropriate emphasis at various grades levels. By having a heading called ‘Ratio and Proportional Relationships’, in Grade 7, Common Core has a chance to emphasize this important middle school topic and highlight connections with this topic. BUT—Common Core does not mention that an equality such as  $2.5/5 = 4.5/9$  is a proportion, and that a student should learn to solve a proportion, such as  $2.5/5 = x/9$ , in which one of the numerators or denominators is unknown. And there is no mention either here or in the geometry section of Grade 7 that one can use proportions, in connection with similarity, to solve for an unknown side of geometric figure—although some would read Item 5 under Geometry to encompass such problems. [I could make the observation at many places in the Common Core document that ‘some might, but many might not, read . . .’ to mean such-and-such. Such ambiguities are troublesome in a working document.]

Suppose I were a middle school teacher or textbook writer looking at the numerical examples under the sub-heading ‘The system of rational numbers’ for Grade 7 on page 35. I see none with fractions or decimals, negative or positive; so I would wonder if numbers like  $-7/4$  and  $-5.36$  should appear in computational problems in Grade 7. Being myself, I think the sub-heading is a clear indication that they definitely should appear in many such problems. It is easy to fix page 35 in order to reflect this view, but I ask myself: Has there

been enough reading and re-reading of the document up to this point in time?

The ‘Expressions and Equations’ section and part of the ‘Functions’ section within Grade 8 of the Common Core draft essentially describe the Grade 8 algebra in the standards. I find the mathematical level to be about right, especially if one takes the view that the items beginning with the word ‘Understand’ also encompass tasks that students should be able to do. In one respect I regard the algebra designated for Grade 8 to be a little too advanced for this grade. Systems of two linear equations should definitely be begun in Grade 8, but to expect proficiency, especially in setting up word problems in two variables, is a stretch.

Item 7 on page 40 in the Geometry section for Grade 8 says: “Explain a proof of the Pythagorean Theorem and its converse.” The word ‘Explain’ invites a description that is vague—a better phrasing would be: “Prove the Pythagorean Theorem and its converse.” However, my opinion is that this is too difficult for Grade 8. [I would be delighted if most eight-graders in the country could verbally distinguish between the Pythagorean Theorem and its converse, leaving the issue of the proofs for high school.]

Item 12 on page 40 in the Geometry section for Grade 8 points to a major problem with the Common Core Standards. This item is prescriptive for teachers, not a desired outcome for students. There are other places in the Common Core draft where a switch in purpose seems to be involved.

Probability and statistics should not be combined (and if they are combined, a much better organization of the items is needed). They have some areas of interrelation just like, say, geometry and proportions have some interrelation. The general vagueness in the descriptions of these areas can give very different impressions to different people about how much emphasis should be given to these areas, especially time spent on projects having more connections to subjects outside of mathematics than to mathematics itself. In general, it is important that the standards avoid open-ended type phrasing that invites rambling by textbook writers and by teachers. This hazard is especially relevant in statistics and probability.

There are 45 items on Functions for high school. The organization and large number of these is an open-invitation for the writing of fragmented, unfocused textbooks. A similar statement applies to high school algebra.

The rather grandiose frontal material could have the effect of convincing school and state administrators that this is a high quality document and thus that they should encourage teachers to follow what is, in my opinion, a fragmented document lacking the necessary coherency.

SUMMARY: I regard the March 2010 Common Core draft to be unsatisfactory. Moreover, I cannot envision any quick fixes. Summary of some major concerns:

- There are too many items (but by this I do not mean too much mathematics).
- Textbooks, written for the purpose of aligning with the Common Core

Standards, will likely be much longer than would be necessary—maybe as much as twice as long.

- Many items beginning with the word ‘Understand’ are open to a wide variety of interpretations.
- The flow of mathematics from grade-to-grade is not as apparent as it should be and the coherence of mathematics within a grade-level is often obscure in the draft.
- The purpose of the standards is not clear—my presumption is that the purpose is to identify mathematical concepts and skills at which students should become proficient in the indicated grades. But neither this nor any other assumption about the nature of the purpose will encompass all the items.
- The draft as a whole does not have the appearance of a well-thought-out document done in a professional manner. This is important. Even in the best of circumstances, it is not easy to get people—in this case, teachers—to welcome a document that places requirements on them and into which they feel they have had minimal input at best. If they can see that the creation of the document has not been given the consideration appropriate for a document that will have such widespread effects, the persuasion becomes even more difficult.